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TWO-PHASE FLOW IN VERTICAL AND INCLINED ANNULI

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Abstract—This paper addresses the problem of estimating void fraction during upward cocurrent two-phase flow in annuli. To model the slip between the phases and the transition between the various regimes, the drift-flux approach is adapted. In analogy to flow behavior in circular conduits, four major flow regimes—bubbly, slug, churn and annular—are recognized. Expressions for void fraction in bubbly, slug and churn flow regimes are derived from the relationship between the phase velocities. The flowing mixture densities calculated from these expressions are useful for pressure gradient calculation. The annular flow regime was not investigated in the present work. The predictions of the proposed method for flow pattern transition and void fraction are compared with experimental data gathered for this work as well as from independent sources. Good agreement between experimental observations and the predictions is noted.

Key Words: two-phase flow, void fraction, vertical/inclined annuli

INTRODUCTION

The importance of multiphase flow to the chemical and petroleum industries has led to proposals of many models and correlations for void fraction and pressure gradient estimation. Most of these studies, however, have been made with circular flow channels. In this work the relationships for void fraction in terms of phase velocities and system properties are developed for two-phase flow through annuli. The upper limit for the flow regime is then established from the hydrodynamic conditions that gives rise to the various flow pattern transitions.

In two-phase flow the static head, $g\rho_m$, is quite often the major contributor to the total head loss, especially for vertical and near-vertical systems. Consequently, an accurate estimation of the gas void fraction, ε (*in situ* volume fraction of the gas), is required because the mixture density, ρ_m, is directly proportional to it. The frictional head loss also requires an estimate of the mixture density and, hence, the gas void fraction, as do such surface properties as interfacial mass and heat transfer coefficients.

This paper first discusses the experimental setup used to gather data, which is followed by the development of expressions for the void fraction in each flow regime. Validation of such developments with our data and those from independent sources is discussed thereafter.

EXPERIMENTAL

A 5.5 m high transparent column was constructed, using a 127 mm i.d. Plexiglas tube to gather data on gas void fraction. Air was fed into a stagnant water column through four side entrances at the bottom of the column. To avoid entrance effects, measurements were taken about 3 m away from the gas inlet. A schematic of the experimental set up is shown in figure 1. Rahman (1984) showed that the entrance effect in this setup diminishes as the pressure drop measurements are made farther away from the inlet. Indeed, measurements at various points in the column (1.83, 2.44 and 3 m away from the inlet) suggested that, other than random errors, there was no difference in the data gathered at the last two stations.

The void fraction in the test section was determined from the pressure drop measured between two points. An estimated frictional component, which never exceeded 1% of the total pressure drop, was subtracted from the total pressure drop before calculating the void fraction.

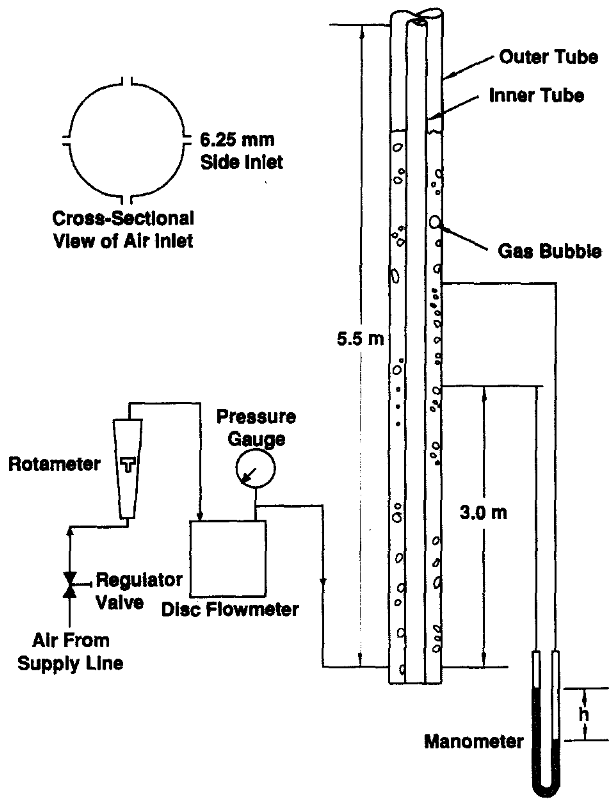

Figure 1. Experimental setup for measuring void fraction.

The superficial air velocity achieved in our setup varied between 0.0066–0.2 m s^{-1}. The system of manometers used allowed measurement of the void fraction down to 0.001. The absolute accuracy of the measurement declined at higher gas flow rates due to manometer fluid level fluctuations. A number of duplicate runs were made, which indicated good reproducibility of the experimental data (Rahman 1984).

Three different inner tubes, with 48, 57 and 87 mm o.d., were used with the outer 127 mm i.d. tube to gather pressure drop data in annular geometry. The bubble rise velocity data, both for small bubbles and large Taylor bubbles, were gathered by determining the time required for bubbles to traverse a 3 m section of the pipe. Taylor bubbles were introduced by suddenly opening and closing an air inlet line. Each bubble rise velocity data set represents an average of 20 measurements. A complete description of the experimental setup and procedure and the accuracy and reproducibility of the data are detailed elsewhere (Rahman 1984; Hasan *et al.* 1988).

In addition to the data gathered from this column, void fraction and flow pattern transition data from several other sources were used in this analysis. Details of the operational conditions for these data are shown in table 1.

Table 1. Experimental conditions of the data used to verify the proposed method

Data source	Diameters (mm)	Fluids	Pressure (kPa)	Temperature (°C)	v_s (m s^{-1})	v_{SG} (m s^{-1})	ϵ
Present work	o.d. = 127 i.d. = 48, 57 & 87	Air Water	Room pressure	Room temp.	0	0.008–0.20	0.01–0.52
Caetano (1984)	o.d. = 76 i.d. = 42	Air Water	200–430	13–40	0.002–3.05	0.0125–14.69	0.04–0.97
		Air Kerosene	320–400	4–26	0.03–2.4	0.029–10.7	0.04–0.97
Sadatomi *et al.* (1982)	o.d. = 30 i.d. = 15	Air Water	Room pressure	Room temp.	0.10–2.00	0.09–15.24	0.1–0.89

BUBBLY FLOW

For bubbly flow, the drift-flux approach provides a simple method for modeling the *in situ* velocity of the gas phase relative to that of the mixture. The *in situ* gas velocity, v_G, is influenced by the tendency of the bubbles to flow through the central portion of the pipe, where the local mixture velocity is greater than the cross-sectional average velocity. In addition, the density difference between the phases gives rise to a drift flux, which adds a velocity equal to the terminal rise velocity of the bubbles, v_∞ (Zuber & Findlay 1965; Hasan 1988a; Hasan & Kabir 1988a,b; Aziz *et al.* 1972), to the lighter phase. Thus,

$$v_G = v_{SG}/\epsilon = C_0 v_m + v_\infty, \qquad [1]$$

where the flow parameter, C_0 is related to the bubble concentration and velocity profiles (Wallis 1969).

Equation [1] may be rearranged to arrive at the following expression for the gas void fraction in terms of the superficial phase velocities:

$$\epsilon = v_{SG}/(C_0 v_m + v_\infty). \qquad [2]$$

The terminal rise velocity depends on the surface tension, σ, and the fluid densities, ρ_L and ρ_G, and appears to be well-represented by the Harmathy (1960) correlation:

$$v_\infty = 1.53[g\sigma(\rho_L - \rho_G)/\rho_L^2]^{0.25}. \qquad [3]$$

If one assumes that the lighter phase flows entirely through the channel center, then it can be shown that the flow parameter, C_0 equals the ratio of the channel center velocity to the cross-sectional average velocity. For turbulent flow, the velocity profile for most of the pipe cross section is quite flat, and the mixture velocity at the axis of the pipe is 1.2 times the average mixture velocity. Although not all the bubbles flow through the central portion of the channel, very few flow close to the wall; and 1.2 has been found to be a reasonable value for C_0 for flow through circular channels.

An exception to this value of 1.2 for C_0 occurs for bubbly flow in large-diameter pipes (>100 mm) with standing liquid columns due to liquid recirculation. In our earlier work (Hasan *et al.* 1988) we reported a value of 2.0 for C_0 for such a system, in agreement with the works of Zahradnik & Kastanek (1979), Haug (1976) and Mashelkar (1970). Clark & Flemmer (1986) presented a slight modification of the drift-flux approach, which suggests a C_0 value of 2.0 for large-diameter stagnant liquid columns.

The analysis leading to [2] also applies to flow through an annular space such as on the shell side of a shell-and-tube heat exchanger or in the tubing/casing annulus of an oil well. It would, however, be necessary to identify any functional dependency of C_0 and v_∞ with the diameters of the channel. To establish this relationship, both terminal rise velocity and void fraction data were gathered with an outer tube and three inner tubes of varying diameters, using air and water as the fluids.

The terminal rise velocity appeared not to be significantly affected by either the inner tubes or the channel deviation from the vertical. The negligible effect of inner tube diameter on bubble rise velocity is not very surprising because the rise velocity of a bubble in an infinite medium [as represented by the Harmathy (1960) equation] has also been found to apply to pipes. The influence of pipe diameter becomes significant only when the diameter of the bubble becomes more than 20% of the channel diameter (Harmathy 1960). Presumably, if we had used a significantly larger inner pipe so as to reduce the annular gap, we may have observed decreased bubble rise velocity. Our earlier work (Hasan & Kabir 1988b) also demonstrated that a pipe deviation of up to 32° from the vertical did not seem to affect the rise velocity of small bubbles. This appears to be the case for annuli as well.

The void fraction data gathered from our experimental setup was then used to determine the effect of various system variables on the flow parameter, C_0.

Effect of pipe inclination

A typical set of the void fraction data gathered is shown in figure 2 for a 127 mm o.d., 87 mm i.d. annular system for channel inclinations of 58° and 90° from the horizontal. The solid line

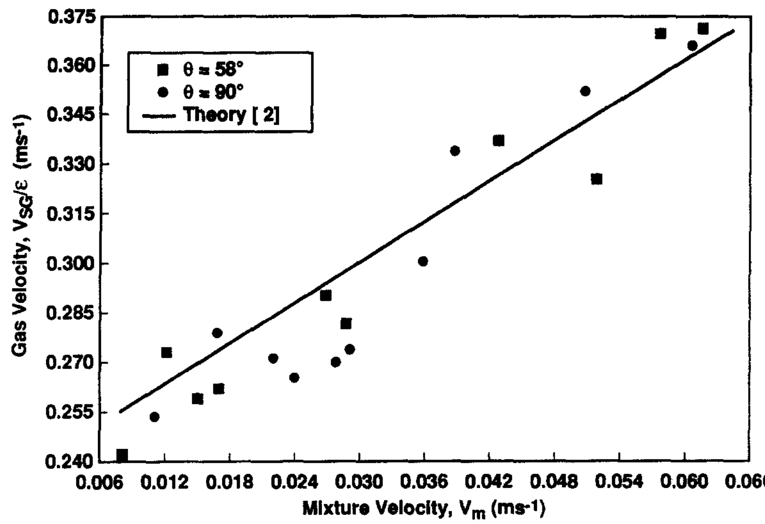

Figure 2. Comparison of theory with experiments in bubbly flow in an air–water system, data from this work.

represents [2] with the rise velocity calculated from the Harmathy (1960) equation and $C_0 = 2.0$. The linear relationship observed between *in situ* gas velocity, v_G, and mixture velocity, v_m, in figure 1, supports the applicability of [1] to annular geometry. Note that the pipe inclination does not appear to affect the void fraction in bubbly flow. We observed in this study, as well as in our earlier work (Hasan & Kabir 1988b; Hasan 1988b), that C_0 in bubbly flow in circular channels is not dependent on the pipe inclination.

Effect of annular dimension

The presence of an inner tube does not appear to influence the bubble concentration profile. The value of C_0 for annuli was found to remain essentially the same as that for a circular channel. This is in contrast to the findings of Hasan & Kabir (1988a) who noted a slight increase in C_0 with the inner-to-outer tube diameter ratio. For the large-diameter system used in this study with stagnant water, C_0 was found to be 2.0, in agreement with our circular channel data and those of Zahradnik & Kastanek (1979), Haug (1976) and Mashelkar (1970). We believe that for smaller tubes (i.d. < 50 mm) the value of C_0 would be 1.2.

Bubbly/slug flow transition

For circular channels, Hasan & Kabir (1988a,b) and Griffith & Snyder (1964) experimentally verified the theoretical contention of Radovich & Moissis (1962) that the transition from bubbly to slug flow occurs at a void fraction, ϵ_t, of about 0.25 in vertical pipes. We found this transition to take place at the same void fraction in annular geometry as well, in agreement with Kelessidis & Dukler (1989). This criterion, $\epsilon_t = 0.25$, may be used to relate the superficial phase velocities using [2] as follows:

$$v_{SG} = (C_0 v_{SL} + v_\infty)/(4 - C_0). \qquad [4]$$

Although for an inclined pipe we found that the terminal bubble rise velocity remained essentially unchanged, [4] requires some modification before it can be applied to the bubbly/slug transition in an inclined channel. In an inclined pipe, the gas phase tends to flow preferentially along the upper wall of the pipe because of gravity. Consequently, the local void fraction at the upper wall exceeds the ϵ_t value of 0.25, even though the cross-sectional average void fraction is well below the transition value.

Assuming that the gas velocity at the upper portion of the pipe is higher by a factor of $1/\sin\theta$ (ratio of actual to projected flow area) compared with the cross-sectional average value, Hasan (1988b) and Hasan & Kabir (1988b) arrived at the following expression for the transition from bubbly to slug flow for inclined systems:

$$v_{SG} = (C_0 v_{SL} + v_\infty)\sin\theta/(4 - C_0). \quad [5]$$

Equation [5] shows good agreement with the data reported by Caetano (1984) for air–water and air–kerosene flow in a vertical annular channel, as shown in figure 3. Even though a good overall agreement is attained between the predictions of [5] and experiments, figure 3 suggests that [5] slightly overestimates the transition v_{SG}, meaning that transition to slug flow occurs at a lower ($\epsilon \sim 0.18$) void fraction in Caetano's system. A similar observation was also made in Caetano's (1984) work. Note that for the air–kerosene system, the bubbly/slug transition boundary lies slightly to the left (i.e. lower v_{SG}) compared with that for the air–water system. The boundary shift is a direct consequence of the 25% lower value of v_∞ for an air–kerosene system compared with that of an air–water system.

The data of Sadatomi *et al.* (1982) for air–water flow in annular and rectangular vertical channels also appear to agree with [5]. In addition, we note that the Kelessidis & Dukler (1989) equation for the bubbly/slug flow transition, supported by their data, is very similar to [5].

Dispersed bubbly flow

At higher flow rates, shear stress caused by turbulence tends to break up the larger bubbles, inhibiting the transition to slug flow even when the void fraction exceeds the value of 0.25. The onset of dispersed bubbly flow, resulting from such dispersion of larger bubbles, was analyzed by Taitel *et al.* (1980) and Shoham (1982). Caetano *et al.* (1992a) adapted this transition criterion for annular channels using the equivalent diameter $D_e (= D_o - D_i)$ and obtained the following expression:

$$2(v_m)^{1.2}(f)^{0.4}(2/D_e)^{0.4}(\rho_L/\sigma)^{0.6}[0.4\sigma/(\rho_L - \rho_G)g]^{0.5} = 0.725 + 4.15(v_{SG}/v_m)^{0.5}. \quad [6]$$

We also recommend the use of [6] for predicting the onset of dispersed bubbly flow in vertical and inclined annuli. At high phase velocities, a characteristic of this flow regime, neither the pipe diameter or the inclination angle is likely to influence the transition to dispersed bubbly flow. Thus, if the mixture velocity is greater than that given by [6], bubbly flow will persist even when the void fraction is >0.25. However, Taitel *et al.* (1980) point out that for small gas bubbles, the gas void fraction cannot exceed a value 0.52. At higher void fractions, transition to slug (or churn) flow occurs. The data of Caetano (1984) show that although [6] overestimates the superficial liquid velocity at which the transition to dispersed bubbly flow occurs in a vertical annulus, the overall agreement is reasonable. An expression similar to [6] was also proposed by Kelessidis & Dukler (1989).

For estimating the void fraction in dispersed bubbly flow, we propose to use [2] with $C_0 = 1.2$ and v_∞ as given by the Harmathy (1960) equation. Dispersed bubbly flow is thus treated similarly to ordinary bubbly flow. A number of researchers have used a homogeneous flow approach in calculating the void fraction and pressure drop in a dispersed bubbly flow regime. However, we believe that although the large superficial velocities involved in dispersed bubbly flow make the fluids more homogeneous, the gas phase still flows mostly through the channel center at a velocity higher than the average mixture velocity.

SLUG FLOW

Application of the drift-flux model to slug flow is more complicated than in bubbly flow because of the different drift velocity of the small bubbles in the liquid slug compared with that of the Taylor bubbles. Assuming that the liquid slugs do not contain any gas bubbles, Hasan (1988a,b; Hasan & Kabir 1988a,b) arrived at the following expression for the gas void fraction using $v_{\infty T}$ for the rise velocity of a Taylor bubble in an annulus:

$$\epsilon = v_{SG}/(C_1 v_m + v_{\infty T}), \quad [7]$$

where C_1 is the flow parameter, analogous to C_0 in bubbly flow. Because the gas phase content in the liquid slug is usually a small fraction of the total gas phase, and the difference in the drift velocities in the slug and the Taylor bubble is usually not very high, Hasan & Kabir (1988a,b) were able to use [7] to correlate void fraction data in slug flow from several sources with good accuracy. Still, using the Taylor bubble rise velocity, which is generally higher than the small bubble rise velocity, for the entire slug flow will cause slight underestimation of the void fraction.

To account for the difference in the drift flux between the liquid slug and the Taylor bubble, a number of rigorous hydrodynamic models have been advanced for circular channels (Sylvester 1987; Fernandes *et al.* 1983). Caetano *et al.* (1992b) have successfully extended this approach to a vertical annular channel. As Vo & Shoham (1989) pointed out, in addition to requiring the solution of a set of eight equations for eight unknowns, such an approach requires an assumption or an empirical relationship for the gas void fraction in the liquid slug.

In this work we simplify the "cellular" approach pioneered by Fernandes *et al.* (1983) for circular channels for adaptation to annuli. Figure 4 shows a model "cell" of length L, consisting of a Taylor bubble of length L_T and a liquid slug of length L_s. Denoting the *in situ* gas fraction in the Taylor bubble portion by ϵ_T and that in the liquid slug portion by ϵ_s, we arrive at the following expression for the average void fraction for the cell:

$$\epsilon = (L_T/L)\epsilon_T + (L_s/L)\epsilon_s. \qquad [8]$$

We use [7] for the Taylor bubble portion of the cell for calculating ϵ_T. Note that this is not strictly valid because [7] is really applicable for an entire cell containing nothing but a Taylor bubble. However, our intention is to approximate the effect of two different drift fluxes within the cell, and not to attempt a rigorous estimation of the contributions of the two portions of the cells separately. Equation [8] may be viewed as a modification of [7], correcting the underestimation of void fraction given by [7].

A number of approaches (Fernandes *et al.* 1983; Sylvester 1987) have been suggested for estimating the liquid slug void fraction, including a recent model presented by Barnea (1990). We adapt a simpler approach by noting that the circular channel data of Akagawa & Sakaguchi (1966) show that the average volume fraction of gas in the liquid slug (based on the total system volume, i.e. $\epsilon_s L_s/L$) is approximately equal to 0.1 when $v_{SG} > 0.4\ \mathrm{m\,s^{-1}}$ and is equal to $0.25v_{SG}$ for lower superficial gas velocities. Assuming that this approximation also applies to annular channels, we rewrite [8] as follows:

$$\epsilon = (L_T/L)\epsilon_T + 0.1 \qquad \text{for } v_{SG} > 0.4\ \mathrm{m\,s^{-1}} \qquad [9a]$$

and

$$\epsilon = (L_T/L)\epsilon_T + 0.25v_{SG} \quad \text{for } v_{SG} < 0.4\ \mathrm{m\,s^{-1}}. \qquad [9b]$$

For $v_{SG} > 0.4\ \mathrm{m\,s^{-1}}$ an expression for the fraction $L_T/L[=1-(L_s/L)]$ may be derived by applying [2] for the gas void fraction in bubbly flow to the liquid slug and noting that it is equal to 0.1. Thus,

$$(L_s/L)\epsilon_s = (L_s/L)v_{SG}/(C_0 v_m + v_\infty) = 0.1 \qquad [10a]$$

or

$$L_s/L = 0.1(C_0 v_m + v_\infty)/v_{SG}. \qquad [10b]$$

Similarly, for $v_{SG} < 0.4\ \mathrm{m\,s^{-1}}$ we obtain

$$L_s/L = 0.25(C_0 v_m + v_\infty). \qquad [11]$$

We point out again that [10a,b] and [11] do not give actual estimates of the relative sizes of the liquid slug and the Taylor bubble. As noted by a number of researchers (Fernandes *et al.* 1983; Sylvester 1987; Caetano *et al.* 1992a,b; Barnea 1990), the length of a liquid slug depends on a number of system parameters and fluid properties. Instead, [10a,b] and [11] in combination with [7] and [9a,b] allow one to calculate the gas void fraction in slug flow, provided values of the flow parameter, C_1, and the Taylor bubble terminal rise velocity are available.

Following the classical work of Nicklin *et al.* (1962), the Taylor bubble rise velocity in vertical circular channels, $v_{\infty T}$, in slug flow is written as

$$v_{\infty T} = C_2\sqrt{gD(\rho_L - \rho_G)/\rho_L} \approx C_2\sqrt{gD}. \qquad [12]$$

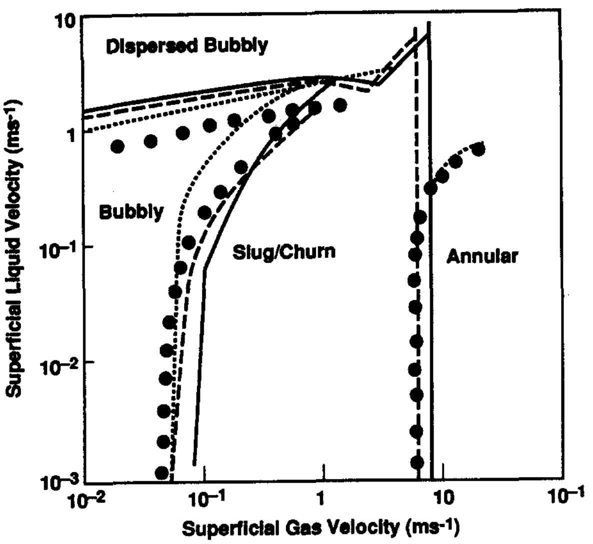

Figure 3. Flow pattern map for vertical flow in an annulus, data of Caetano *et al.* (1992a). —— Theoretical, water; ●●● Experimental, kerosene; --- theoretical, kerosene; ------ experimental, water.

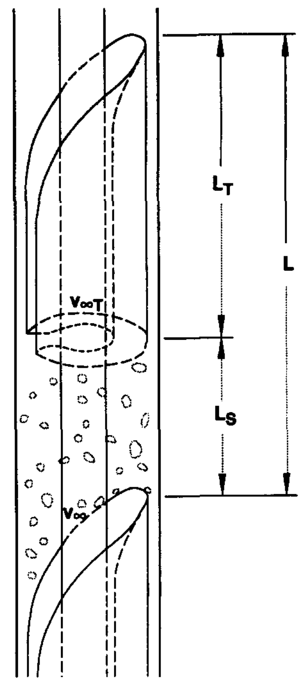

Figure 4. Schematic of a model "cell" in slug flow in an annulus.

Extensive data and theoretical analyses by a number of researchers indicate that, although influenced by the forces of inertia, viscosity and surface tension, the values of C_2 remains constant at 0.345 for many practical systems. Barnea & Shemer (1986) have provided theoretical justification for this experimental value for C_2, as obtained by Griffith (1964) and Sadatomi *et al.* (1982).

For slug flow in inclined circular channels, Bendiksen (1984), Hasan (1988b) and Hasan & Kabir (1988b) found that the value of C_1 remained constant at 1.2. However, they observed significant variation in the terminal rise velocity of a Taylor bubble with deviation of the pipe from the vertical. As the pipe is deviated from the vertical, the nose of the Taylor bubble becomes sharp, causing a reduction in the drag force with a consequent increase in the rise velocity. However, when the pipe is highly deviated, the reduction in the buoyancy force offsets the reduction in the drag force, causing a decrease in the rise velocity compared with that for a vertical pipe. The variation in the Taylor bubble rise velocity with the pipe inclination was given by Hasan & Kabir (1988b) and Hasan (1988b) in the following manner:

$$v_{\infty T\theta} = v_{\infty T}\sqrt{\sin\theta}\,(1+\cos\theta)^{1.2}. \qquad [13]$$

Hasan & Kabir (1988b) noted good agreement of [13] with the circular channel data of Runge & Wallis (1965) for most inclination angles, the exceptions being data from pipes inclined by $\leqslant 20°$ from the horizontal. Note that [13] predicts zero rise velocity for Taylor bubbles in horizontal systems, while some researchers have noted a nonzero rise verocity in horizontal pipes (Weber *et al.* 1986). Other approaches for estimating $v_{\infty T\theta}$ in inclined pipes are available (Maneri & Zuber 1974; Bendiksen 1984). However, because of the reduced importance of void fraction in calculating the pressure drop in near-horizontal systems, refining $v_{\infty T\theta}$ estimation for such systems is perhaps unwarranted. Indeed, Hasan & Kabir (1988b) showed good agreement between experiment and theory for both void fraction and pressure drop data for inclination angles of $\geqslant 20°$ in circular channels. Thus, we propose a limiting inclination angle of 20° for [13]. In the following, we discuss the effects of annular dimension and pipe inclination on the flow parameter and the Taylor bubble rise velocity as given by [13].

Effect of annular dimension

The approach used in bubbly flow to determine the effect of annular dimension and pipe inclination on the flow parameter can only be used in slug flow if [7] alone is used to estimate the gas void fraction. In such a case, v_{SG}/ϵ has a linear relationship with v_m, with a slope of C_1. Although imprecise, this approach indicates that the flow parameter, C_1, is not significantly influenced by either inner tube diameter or pipe inclination. Thus, we propose to use a constant value of 1.2 for C_1 for all cases of slug flow. We note that Hasan & Kabir (1988a) found C_1 to vary slightly with the inner-to-outer pipe diameter ratio. Use of a different expression for the rise velocity in the earlier work probably explains the difference.

The presence of an inner tube tends to make the nose of the Taylor bubble sharper, causing an increase in the rise velocity, $v_{\infty T}$. The Taylor bubble rise velocity data gathered for the present work agreed with the suggestion of Griffith (1964) that the diameter of the outer tube should be used in [13] for estimating $v_{\infty T}$ in the annulus. Our Taylor bubble rise data show a linear relationship with the diameter ratio, D_i/D_o, suggesting the following expression for the Taylor bubble rise velocity for vertical annular systems:

$$v_{\infty Ta} = [0.345 + 0.1(D_i/D_o)]\sqrt{gD_o(\rho_L - \rho_G)/\rho_L}. \qquad [14]$$

Effect of pipe inclination

The effect of pipe inclination on the rise velocity appears to be well-represented by the correlation for circular channels, [13]. Thus, combining [13] and [14], we arrive at the following expression for the Taylor bubble rise velocity in inclined annuli:

$$v_{\infty T\theta a} = [0.345 + 0.1(D_i/D_o)]\sqrt{\sin\theta}\,(1 + \cos\theta)^{1.2}\sqrt{gD_o(\rho_L - \rho_G)/\rho_L}. \qquad [15]$$

Figure 5 shows a good agreement of the Taylor bubble rise velocity predicted by [15] (——) with our data. However, this figure also suggests that when the channel is highly deviated from the vertical, [15] appears to overestimate the effect of inclination. Because our system could not be deviated by more than 32° from the vertical, no attempt was made to modify [15] further to account for this overestimation.

We note with interest that Griffith (1964) also observed a similar variation in $v_{\infty Ta}$ with annular diameter, although his data indicate a weaker dependence of this parameter on D_i/D_o. In contrast, Sadatomi *et al.* (1982) presented data and a general approach for estimating the Taylor bubble rise velocity in various noncircular vertical channels, in terms of the equiperipheral diameter, D_{ep}, as follows:

$$v_{\infty Ta} = 0.345\sqrt{gD_{ep}(\rho_L - \rho_G)/\rho_L}. \qquad [16]$$

The equiperipheral diameter is defined as the wetted perimeter of the channel divided by π, which is equal to the sum of the annuli diameters, $D_i + D_o$, for an annulus. Agreement of their correlation with their data from annuli is less satisfactory than with data from other channels. The average

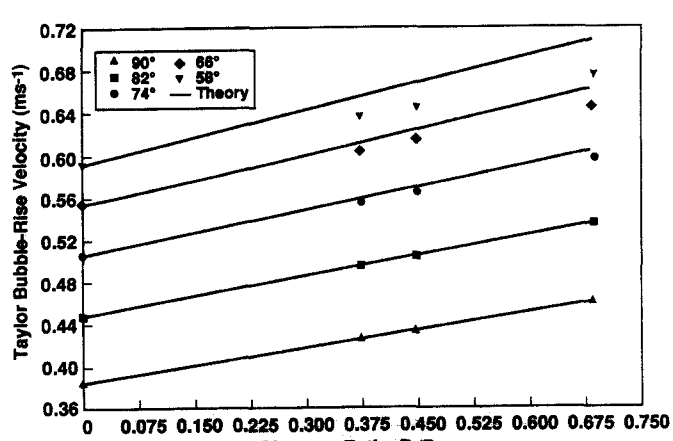

Figure 5. Behavior of the Taylor bubble velocity in inclined annuli, data from this work.

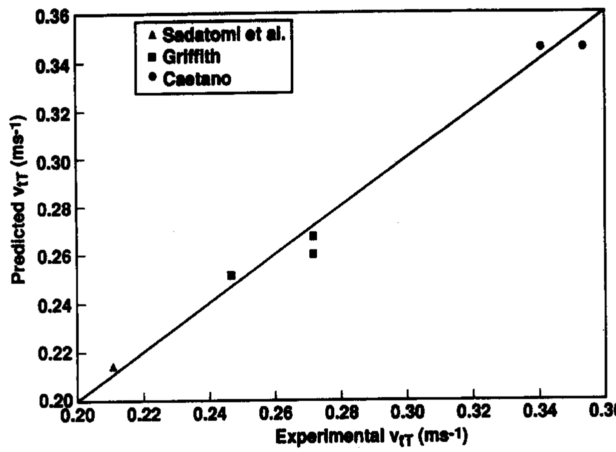

Figure 6. Predicting the Taylor bubble velocity in vertical annuli.

errors in predicting the $v_{\infty Ta}$ data of Caetano (1984) for the Sadotomi *et al.* (1982) correlation are +6.8% for the air–water system and +10.52% for the air–kerosene system. Our correlation [15] predicted the same data with average errors of −2.09 and +1.46%, respectively.

For estimating the Taylor bubble rise velocity in vertical systems, $v_{\infty Ta}$, Hasan *et al.* (1988) recommended using [14] with the equivalent diameter, $D_e(=D_o - D_i)$, in place of the outer pipe diameter, D_o. However, they did not actually measure the Taylor bubble rise velocity. Instead, they based their correlation on the best fit for their void fraction data to [7]. The lower estimate of the bubble rise velocity was balanced by an increase in C_1 with the diameter ratio, D_i/D_o, leading to a reasonable prediction of the void fraction. In view of the observed increase in the Taylor bubble rise velocity in an annulus, however, the expression proposed here is to be preferred.

The predictions using [14] of the rise velocity data reported for vertical annular channels by Caetano (1984), Sadatomi *et al.* (1982) and Griffith (1964) are shown in figure 6. The agreement appears to be quite good.

Slug/churn flow transition

Because of the chaotic nature of churn flow, gathering data for this transition as well as modeling the transition is subject to error. Models based on the "flooding" phenomenon appear to be inapplicable at high pressures (Hasan & Kabir, 1988a). Moreover, Weisman & Kang (1981) showed the discrepancy in data and empirical correlations from a number of sources. Therefore, no attempt was made to delineate this transition for annular geometry. Instead, we suggest that the model proposed by Brauner & Barnea (1986) for circular channels be used for the transition to churn flow even for annuli. This criterion suggests that the transition to churn flow occurs when the gas void fraction in the liquid slug following the Taylor bubble exceeds 52%. The Brauner–Barnea approach may be superior to the other correlations as it appears to account for the effect of the pipe inclination on this transition. Barnea *et al.* (1985) and Brauner & Barnea (1986) have noted that even a slight deviation of the pipe from the vertical greatly reduces the occurrence of churn flow and that the flow regime vanishes entirely when the pipe is deviated by more than 20° from the vertical.

CHURN FLOW

The churn or froth flow pattern has not been investigated extensively because of its chaotic nature. However, the analyses presented for bubbly and slug flow should also be applicable for the churn flow pattern. Thus, the equations developed for predicting void fraction in slug flow ([7]–[11]) are used for the churn flow regime. Equation [7] was also suggested by Fernandes *et al.* (1983) for circular channels and by Kelessidis & Dukler (1989) for annuli. Although the bubble shape is quite different from the classical Taylor bubble, the bubble rise velocity during churn flow is probably not much different from that for slug flow. In addition, because the mixture velocity is much higher than the bubble rise velocity during churn flow, a slight error in estimating $v_{\infty T\theta a}$ does not significantly affect void fraction estimation.

On the other hand, an accurate estimation of C_1 is very important for predicting void fraction. The bubble concentration profile in churn flow may be dissimilar to that for slug flow because of the characteristic churning motion of this flow regime. Using [7] with $v_{\infty T}$ given by [12], Hasan (1988a,b) analyzed the void fraction data, gathered by Ney (1968) and Fuentes (1968) for vertical circular channels in the churn flow regime, and concluded that a value of 1.15 was appropriate for the parameter C_1. We, therefore, propose to use [8] for estimating the void fraction in churn flow in annuli with $C_1 = 1.15$ and $v_{\infty Ta}$ as given by [15] in an annular system.

Transition to annular flow

The transition from churn (or slug) to annular flow was not investigated experimentally in the present work. Instead, the approach adapted by Taitel *et al.* (1980) for vertical circular channels, based on the drag force necessary to keep the entrained liquid droplets in suspension during annular flow, is used here for annular geometry as well. At the high velocities associated with annular flow, Weisman & Kang (1981) and Barnea *et al.* (1985) observed that the orientation of the system has little effect on the transition to annular flow. We extend the concept to annular geometry and propose using the Taitel *et al.* (1980) equation for the transition to annular flow. Thus,

$$v_{SG} = 3.1\sqrt[4]{\sigma g(\rho_L - \rho_G)/\rho_G^2}. \qquad [17]$$

Figure 3 shows good agreement between [17] and the experimental data gathered by Caetano (1984) for both air–water and air–kerosene vertical systems. Because the system operating pressure and temperature have a profound impact on the fluid properties of density and interfacial tension, the transition superficial velocity as given by [17] will change markedly. Consequently, the transition boundaries, shown in figure 3, would move depending upon the operating conditions. Such movement of boundaries was illustrated by Taitel *et al.* (1980) for two different system pressures.

We note that the complicated model for the transition to annular flow presented by Kelessidis & Dukler (1989) is very close to that proposed by [17], as was pointed out by Kelessidis (1986). Thus, we retain [17] for simplicity.

COMPARISON WITH PUBLISHED DATA

Published gas void fraction for two-phase flow through annular geometries are scarce. Caetano (1984) gathered void fraction, pressure drop and flow pattern data for air–water and air–kerosene two-phase flow through an annulus with a 72.6 mm i.d. outer tube and a 42.16 mm o.d. inner tube at room temperature and a pressure of about 3 atm. We compare the predictions of our method against this data set. Note that the flow pattern observed by Caetano *et al.* (1992a) is used here.

Table 2 presents statistical information regarding this comparison in terms of the average error and standard deviation in the prediction of data in each flow regime. The average error, $\bar{e}$, is defined as the sum of the errors, e_i, divided by the number of data points, n ($\bar{e} = \Sigma\, e_i/n$). The error is the difference between the predicted and experimental values of the liquid holdup, $1-\epsilon$. Thus, a positive

Table 2. Statistical comparison of the proposed method's predictions using Caetano's (1984) data

			Slug		
Flow regime	Bubbly	Dis. bubbly	[7]	[9a,b]	Churn
		Air–Water Data			
Average error	0.022	0.043	−0.028	0.041	−0.005
Average % error	2.400	5.000	−6.450	7.040	[a]
Std dev.	0.040	0.049	0.072	0.063	0.033
% Std dev.	4.070	5.020	12.700	8.100	[a]
		Air–Kerosene Data			
Average error	0.013	0.003	0.052	0.047	0.026
Average % error	1.900	0.600	10.900	9.930	[a]
Std dev.	0.034	0.018	0.061	0.057	0.043
% Std dev.	3.980	2.160	6.900	7.200	[a]

[a] Average percentage error and percentage standard deviation are not computed for the data in churn flow because of the very low liquid holdup values of some of the data in this regime.

average error indicates overestimation of the liquid holdup by the proposed method. The standard deviation, σ, is defined as the square root of the sum of the squares of errors divided by (number of data − 1) [$\sigma^2 = (\Sigma e_i^2)/(n-1)$].

Figure 7 presents the liquid holdup predictions of the proposed method for the bubbly flow data of Caetano (1984) for the air–water and the air–kerosene systems. General overestimation of the liquid holdup is evident for both fluid systems, although agreement is much better for the air–kerosene system.

The air–water bubbly flow data were predicted with an average percentage error, defined as $(1/n)100\,\Sigma\,(e_i/(1-\epsilon_{\text{experiment}}))$, of 2.4%. The percentage standard deviation, defined as $\sigma_p = 100\,\Sigma\,(e_i - \bar{e})^2/(n-1)$, is 4.07% for these data. The model proposed by Caetano *et al.* (1992b), which contains a parameter optimized by using the data set, predicted the same data with an average percentage error and percentage standard deviation of −0.46 and 4.75%, respectively. Two data points in this set are suspect because the liquid holdup for these data points are actually lower than the value the homogeneous model would predict, suggesting that the liquid has a higher *in situ* velocity than the gas. Our method predicted the air–kerosene bubbly flow data of Caetano with an average percentage error of 1.90% and a percentage standard deviation of 3.98% compared with −2.17 and 5.21%, respectively, for the Caetano *et al.* (1992b) model. In this set there was also at least one datum that suggested a higher liquid *in situ* velocity than that of the gas phase. The general overestimation suggests either a lower value of C_0 or a lower v_∞ for annuli.

The dispersed bubbly flow data of Caetano (1984) and their predictions by the proposed method are shown in figure 8. The air–water liquid holdup data in this flow regime are also generally overestimated, on average by 5.0% compared with an underestimation by 1.67% if the homogeneous model is assumed (i.e. $\epsilon = v_{SG}/v_m$), as proposed by Caetano *et al.* (1992b). However, as pointed out earlier, treating the dispersed bubbly flow as homogeneous may not be sound because it assumes a value of 1.0 for C_0 when most of the bubbles still flow through the core of the channel. The air–kerosene data of Caetano appear to support this contention; our method predicts these data with an average error of 0.60% and a standard deviation of 2.16% compared with −3.84 and 2.93% for the homogeneous model.

The slug flow liquid holdup data of both air–water and air–kerosene systems, shown in figure 9, are also overestimated by the proposed method. Our method ([9a,b]) overestimates the air–water slug flow data by 7.04% and the air–kerosene data by 9.93%, while percentage standard deviations for these two sets of data are 8.1 and 7.2%, respectively. These higher values of the percentage errors reflect the generally lower value of liquid holdup, rather than any inaccuracy in predicting the absolute value as indicated in table 2.

As table 2 shows, the predictions of the simplified approach ([17]) suggested by Hasan & Kabir (1988a,b), in which all the gas is assumed to have the same drift flux as that of the Taylor bubbles, are very similar to the predictions of the method proposed here. Considerations of the different drift fluxes in the Taylor bubble and the liquid slug, as proposed here, reduced the standard deviation by only 0.01 for the air–water data and by 0.004 for the air–kerosene data. However, the rigorous slug flow model developed by Caetano *et al.* (1992b) predicted the liquid holdup data for both systems somewhat better. Caetano *et al.* predicted the air–water slug flow data with $\bar{e}_{ip} = 3.94\%$ and $\sigma_p = 7.72\%$; and the air–kerosene data with $\bar{e}_{ip} = 5.8\%$ and $\sigma_p = 9.97\%$. This superior prediction is partly due to the fact that the model proposes separate approaches for developed and developing Taylor bubbles, and that the model was developed from the Caetano data set. Holdups of 80 and 85% in the liquid slug for the air–water and air–kerosene systems, as used by Caetano *et al.* (1992b) may not always be applicable.

The comparison for the churn flow data is shown in figure 10. The low value of the average error supports the lower value for the parameter, C_1, as proposed in [7]. Average percentage error and percentage standard deviation are not reported for these data because liquid holdup values of many of the data in this flow regime are very low, resulting in percentage errors that exaggerate the inaccuracy of the method. No correlation was proposed by Caetano *et al.* (1992b) for this flow regime.

Figure 11 compares the predictions of the proposed method with the air–water data Sadatomi *et al.* (1982) collected, using a 30 mm o.d, 15 mm i.d. annulus. We predicted the gas void

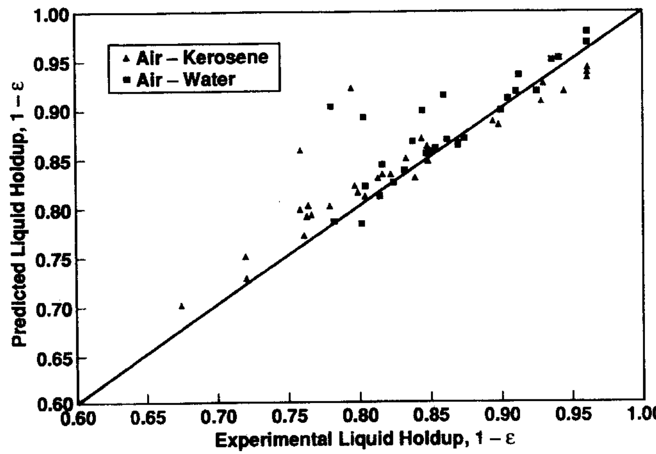

Figure 7. Predicting the liquid holdup in bubbly flow, data of Caetano *et al.* (1992b).

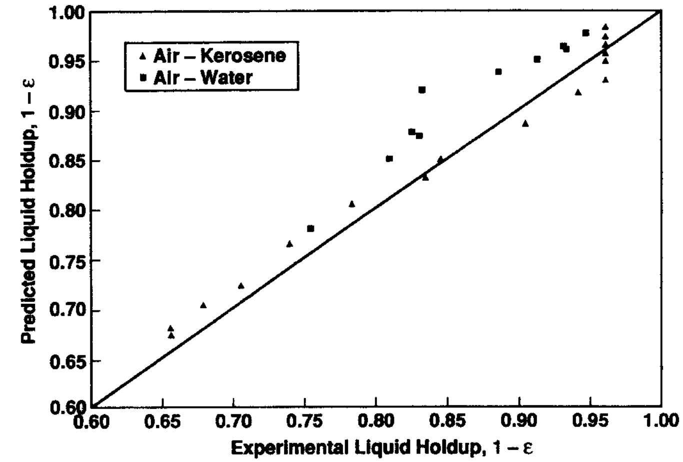

Figure 8. Predicting the liquid holdup in dispersed bubbly flow, data of Sadatomi *et al.* (1982).

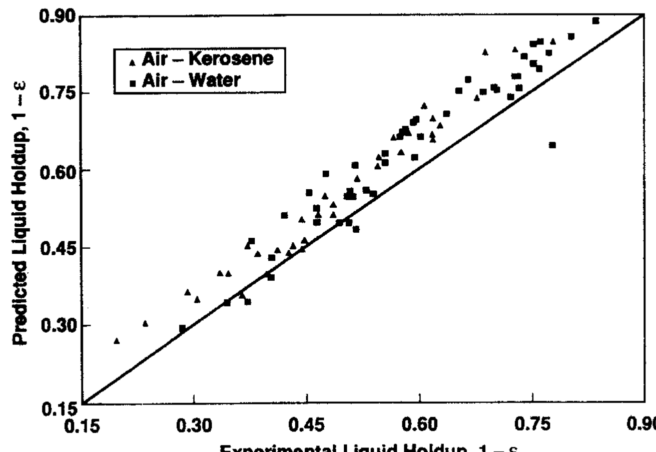

Figure 9. Predicting the liquid holdup in slug flow, data of Caetano *et al.* (1992b).

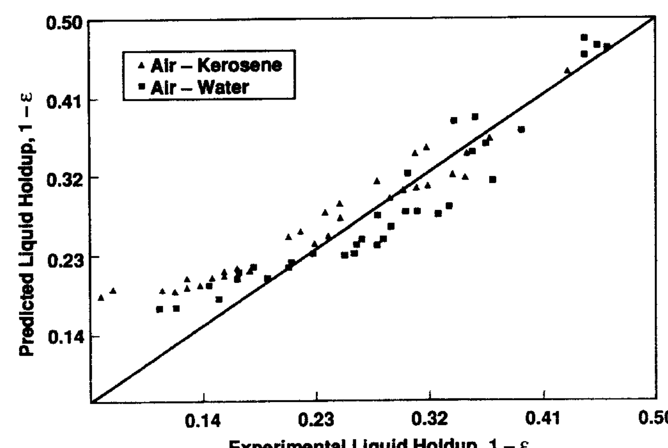

Figure 10. Predicting the liquid holdup in churn flow, data of Caetano *et al.* (1992b).

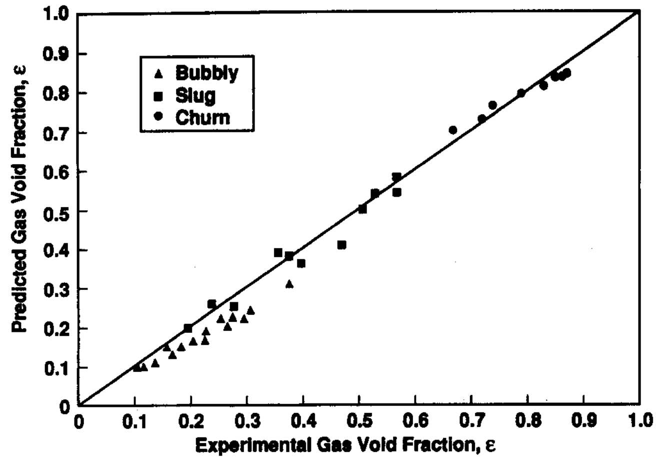

Figure 11. Predicting the void fraction in bubbly, slug and churn flow, data of Sadatomi *et al.* (1982).

fraction in all flow regimes in this data set with an average error of 0.023 and a standard deviation of 0.0214.

A good agreement between the Sadatomi *et al.* (1982) data and our predictions can be noted in figure 11 in the slug and churn flow regimes. The bubbly flow void fraction data are slightly underestimated (i.e. liquid holdup is overestimated), as in the case of the data of Caetano (1984). A lower value of the terminal rise velocity of small bubbles (about 0.08 m s^{-1} instead of 0.24 m s^{-1} as used in this analysis) would make the predictions agree very well with the data. This would also be true of the bubbly flow data of Caetano (1984). It is possible that for the small-diameter pipes used by Sadatomi *et al.* (1982), the terminal bubble rise velocity of small bubbles is actually lower, perhaps being affected by the pipe walls. However, Sadatomi *et al.* did not provide small-bubble rise velocity data to verify this point. In addition, the annulus used by Caetano (1984) is too large for the above argument to be valid, although Caetano did not provide small-bubble rise velocity data either. Additional data with varying annular dimension are needed to clarify this point.

DISCUSSION AND CONCLUSION

This paper presents the flow pattern approach to predicting the void fraction in bubbly, slug and churn flow regimes in both vertical and inclined annuli. Transitions from one flow regime to another are also discussed. The method is based on the relative motion between the gas and liquid phases, caused by the density difference between the phases and the tendency of the gas phase to flow through the central portion of the channel. The terminal bubble rise velocity accounts for the buoyancy effect, while the effect of the bubble concentration profile is accounted for by using the flow parameters, C_0 and C_1.

The terminal rise velocity for bubbly flow appears to be unaffected by annular geometry and was well-represented by the Harmathy (1960) equation. However, the data of Caetano (1984) and Sadatomi *et al.* (1982) suggest a somewhat lower value for the bubble terminal rise velocity. The Nicklin *et al.* (1962) correlation was found to be adequate for the Taylor bubble rise velocity in the slug and churn flow regimes when the increase in the rise velocity with the inside-to-outside diameter ratio of the annulus and the inclination of the channel was accounted for.

The flow parameters in bubbly, slug and churn flow, C_0 and C_1, appear to be unaffected by the annular dimension. Values of C_0 and C_1 appropriate for circular channels are, therefore, recommended for annuli.

The transition from bubbly to slug flow was observed to occur at a void fraction of about 0.25 for both annular and cylindrical geometries. The difficulty in predicting the transition to churn flow was pointed out. Use of model proposed by Taitel *et al.* (1980) for the transition from churn to annular flow was suggested in this work.

The proposed method was compared with data from several sources. The good agreement between the data and the predictions lends support to the soundness of the method presented.

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